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Olivier Piguet Silvio P. Sorella

Algebraic Renormalization

Perturbative Renormalization, Symmetries and Anomalies

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Preface

The idea of this book originated from two series of lectures given by us at the Physics Department of the Catholic University of Petrópolis, in Brazil. Its aim is to present an introduction to the "algebraic" method in the perturbative renormalization of relativistic quantum field theory. Although this approach goes back to the pioneering works of Symanzik in the early 1970s and was systematized by Becchi, Rouet and Stora as early as 1972–1974, its full value has not yet been widely appreciated by the practitioners of quantum field theory. Becchi, Rouet and Stora have, however, shown it to be a powerful tool for proving the renormalizability of theories with (broken) symmetries and of gauge theories. We have thus found it pertinent to collect in a self–contained manner the available information on algebraic renormalization, which was previously scattered in many original papers and in a few older review articles.

Although we have taken care to adapt the level of this book to that of a post-graduate (Ph.D.) course, more advanced researchers will also certainly find it useful.

The deeper knowledge of renormalization theory we hope readers will acquire should help them to face the difficult problems of quantum field theory. It should also be very helpful to the more phenomenology oriented readers who want to familiarize themselves with the formalism of renormalization theory, a necessity in view of the sophisticated perturbative calculations currently being done, in particular in the standard model of particle interactions.

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Geneva and Rio de Janeiro,
February 1995 *Olivier Piguet and Silvio P. Sorella*

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1. Introduction

Relativistic quantum field theory (RQFT) was introduced at the end of the 1920s in order to unify the principles of quantum mechanics and of special relativity (see for example the first of Dirac's papers in [1]). Since that time RQFT has been the object of ever increasing interest. Together with some of its extensions, such as string theory, it has now become our main conceptual framework for describing the fundamental structure of matter [2]. We underline, in particular, the well established and successful applications to the gauge theories of particle interactions: QED, the standard model, QCD and supersymmetric extensions thereof. Although a quantum theory of gravity has not yet been firmly formulated, there are now hopes for a consistent field theoretical description as well [3].

However, it has been realized since its beginnings (see [4] and Heisenberg's paper in [1]) that RQFT was plagued by an apparently insuperable difficulty, namely the problem of the ultraviolet (UV) divergences, which had forbidden any computation beyond the lowest order of perturbation theory. The origin of this difficulty lies in the occurrence of singular products of distributions at coinciding points in the computation of the higher–order terms [5, 6]. As is now well known, the search for a solution to the problem of the subtraction of the UV divergences, lasting from the 1940s to the 1970s, has lead to the establishment of a consistent theoretical and mathematical framework called renormalization theory [1, 7, 8]. These efforts, rewarded first by the spectacular agreement of theory with experiment in QED [1, 9], culminated in the proof of the renormalizability of the nonabelian gauge theories [10, 11], which are the basis of our present understanding of high–energy physics.

At the beginning of the 1970s a few UV subtraction schemes were available and were proven to be equivalent [7]. It is a remarkable result that these schemes led to a renormalized perturbation theory whose properties can be collected in a set of rigorous statements, common to all of them. These statements, based on locality and power counting, constitute the content of the so–called quantum action principle (QAP) [12, 13, 14]. The latter is the starting point of any rigorous investigation in the framework of perturbation theory.

It was early recognized by Becchi, Rouet and Stora, in their works on the BRS invariance of gauge theories [15, 16] and on the models with broken rigid symmetries [17], that the use of the QAP leads to the possibility of a fully algebraic proof of the renormalizability of a theory characterized by a set of local or rigid invariances. Let us remark that we distinguish "renormalization", which for us means the implementation of the symmetries of a given classical model to all orders of perturbation series, from the mere "UV subtractions". The QAP actually allows one to control the breaking of a symmetry induced by a noninvariant subtraction scheme, helping then