Mathematical Engineering

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Bubble Systems

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Preface

The main purpose of this book is to bring together into a coherent system all our knowledge about the physical mechanisms governing the laws of behaviour of both the simplest bubble systems consisting of single bubbles and of involved systems, which occur, for example, in bubble flows or during bubbling.

Due to the rapid progress in science, both textbooks and scarce scientific monographs, partially embracing this field of knowledge, tend to age rather quickly, often losing their actualité right after publication. I would like very much that this book does not share such a fate. This wish has determined its structure.

First, we give a systematic treatment of the fundamental problems describing the dynamics of single bubbles (laws of their growth, collapse, fragmentation and emersion in the gravitational field, pulsation and so on.). The use of modern software products for analytical studies helps to obtain new quite surprising results even for these classical problems, whose authors in due time remarkably combined deep physical sense, serious mathematical technique, and enormous amount of computational and analytical work to obtain rigorous analytical solutions. Moreover, despite the classical nature of the problems under consideration, for some of them the complete analytical solutions were obtained only in the past two decades and at present are unknown to the broad research community.

Second, based on these fundamental solutions it is possible to put forward, in a large number of cases, fairly rigorous analytic solutions of next-level problems, which are traditionally farmed out to experimentalists or specialists in the field of computational fluid dynamics. As example, we may mention the problems on hydrodynamics of bubble flows in channels, sizes of bubbles, true void fraction, hydraulic resistance during boiling of subcooled liquid under forced motion conditions, and so on. Despite the fact that in a number of respects we cannot avoid using the numerical methods, the particular analytic solutions, as well as the recourse to qualitative methods of investigations residing in consistent application of similarity methods, enable one to carry out the results of analysis to design relations, which secure the corresponding passages to the limit and have wide range of applications and high accuracy.

Third, from the methodological point of view, of great value are the conceptual developments in the field of bubble flows, and in particular, the theory of boiling shock, which was created by the author in the 1980s, and the applications of the Reynolds analogy in the study of nonequilibrium two-phase flows.

The systematic use of the theory of boiling shock has enabled one to explain, and in many cases to predict, quite a number of experimental effects accompanying the processes of discharge of flashing liquid, the processes of unsteady flashing with rapid pressure relief, and to build on its basis methods for calculation of similar phenomena.

Reynolds analogy for flows with ‘double disequilibrium’ (superheated near-wall layer of liquid—subcooled liquid in the core), which often occur in high-performance cooling systems, proved a fairly natural tool in the study of processes of radial transfer of heat and momentum, providing thereby a basis for obtaining fairly rigorous analytic solutions of the heat exchange and hydrodynamics of such flows.

There are already thousands of scientific and engineering papers relating to nonequilibrium two-phase flows. Frequently, these studies consider the same phenomena from quite different (and in a number of cases from directly opposite) viewpoints. In some cases (fortunately, its number is quite narrow), erroneous approaches migrate from one paper to another to gradually become traditional.

Despite the fact that this branch of science reached its full flowering in the 1970–80s due to the rapid development of nuclear and aerospace engineering, the research activity in this field has been quite intensive up to now, which can be explained by its operational and scientific significance. Consequently, a young scientist, who has received a good fundamental background and begins his or her study in this field, will only after several years be able to orientate in this multi-dimensional space, to be capable of distinguishing the important tendencies of the theory from the dead-end tracks.

A research engineer embarking on a study of nonequilibrium two-phase systems should be familiar with numerous classical disciplines like the thermodynamics and the theory of the properties of matter, the mechanics of liquid, gas dynamics as well as the theory of heat and mass transfer. General textbooks and handbooks on these scientific disciplines contain such a huge amount of information that they cannot always be comprehended and made valuable by an amateur in thermal physics. A serious problem for a student is the choice of subdisciplines that are vital in his or her studies.

In this process, contacts with elder colleagues and supervisors are invaluable. Maturing of a young scientist is believed to be additionally facilitated by this book, which includes the principal achievements in the construction of the theory and illustrates application of the principal tools for its further development.

In writing this book, the author was mainly focused on the demands of graduate and postgraduate students specializing in two-phase flows physics. This book will be a useful complement to existing textbooks on heat exchange and gas dynamics, with the aim toward in-depth study of these disciplines and providing help in comprehending the fundamentals of the theory of bubble systems.

The true needs of practicing professionals and postgraduate students are not far apart. Both require, above all, an understanding of the fundamentals of running processes, which is the main purpose of this book.

All solutions, without exception (both analytic and numerical), obtained in this book expand to fairly simple design formulas, many of which are put forward for the first time. Hence, this book also serves as a detailed handbook that can be conveniently used in analytical, numerical and design practice.

In conclusion, the author expresses his sincere gratitude to his numerous colleagues, coworkers and friends, whose discussions and valuable comments were instrumental in bringing the book to its final form.

The author is deeply indebted to Prof. Bernhard Weigand (Stuttgart University), whose courteous attention and support encouraged him in this endeavour.

A great role in the forming of the author's entire scientific outlook was played in due time by his supervisor Prof. Dmitry Labuntsov (1929–1992). The fact that the reader takes this book in his or her hands means that Labuntsov's ideas continue to develop and his blessed memory will live on.

Moscow Alexander A. Avdeev
December 2015

Contents

Symbols

A	Amplitude of pulsations
a	Amplitude of the surface pulsations, m
$b = \sqrt{\frac{\sigma}{g(\rho_l - \rho_v)}}$	Capillary constant, m
C	Phase velocity of the wave
C_D	Drag coefficient
c	Speed of the sound, m/s
$c_f = \frac{2\Delta p}{\rho w^2}$	Friction factor
c_p	Specific heat capacity at constant pressure, J/(kgK)
c_v	Specific heat capacity at constant volume, J/(kgK)
c_μ	Mean velocity of thermal motion of molecules, m/s
D	Diffusion coefficient, m^2/s
d	Diameter, m
d_0	Mean roughness size, m
$d_{32} = \frac{d_V^3}{d_F^2}$	Sauter mean diameter of bubble, m
d_b	Diameter of bubble, m
$d_e = \frac{4F}{\Pi}$	Equivalent diameter of the channel, m
$d_F = \sqrt{\frac{F}{\pi}}$	Surface averaged diameter of bubble, m
$d_V = \sqrt[3]{\frac{6V}{\pi}}$	Volume averaged diameter of bubble, m
E	Energy, J
e	Specific internal energy, J/kg
e_t	Energy flow of turbulent dissipation, W/kg
F	Square, m^2
f	Frequency, Hz
f_D	Size distribution function, 1/m
f_V	Volume distribution function, $1/\mathrm{m}^3$
G	Mass rate, kg/s
g	Gravitational acceleration, $\mathrm{m/s}^2$

H	Mean curative of the surface, 1/m
h	Specific enthalpy, J/kg
$h_* = h + \frac{w^2}{2}$	Stagnation enthalpy, J/kg
h_{fg}	Latent heat of evaporation, J/kg
I	Volume onset of the property
I_F	Rate of surface nucleation sites, $1/(\mathrm{m}^2\mathrm{s})$
I_V	Rate of volume nucleation sites, $1/(\mathrm{m}^3\mathrm{s})$
$\vec{J}$	Flow density of the property
j	Mass flux density (mass velocity), kg/(m^2s)
$k = \frac{2\pi}{\lambda}$	Wave number, 1/m
$k_B = \frac{R_\mu}{N_A}$	Boltzmann constant, J/K
k_q	Heat transfer coefficient W/(m^2K)
k_m	Mass transfer coefficient kg/(m^2s)
k_s	Equivalent height of sand roughness m
k_t	Thermal conductivity, W/(m^2K)
l	Length scale, m
$l_* = \frac{\nu}{\sqrt{\tau_w/\rho}}$	Friction length, m
m	Bubble growth modulus
N	Number of molecules per unit of liquid volume, $1/\mathrm{m}^3$
N_A	Avogadro constant, 1/mol
n_F	Surface density of active nucleation sites, $1/\mathrm{m}^2$
p	Pressure, Pa
Q	Quantity of heat, J
q	Heat flux density, W/m^2
q_V	Volume heat source, W/m^3
R	Radius of bubble, m
R_0	Initial bubble radius, m
$R_* = \frac{2\sigma}{\Delta p}$	Critical bubble radius
$R_e = 0.5 d_V$	Equivalent radius of a bubble, m
R_g	Specific gas constant, J/(kgK)
R_S	Radius of the spherical cap, m
R_μ	Universal gas constant, J/(molK)
r	Radial coordinate, m
r_0	Radius of the channel, m
r_e	Equivalent radius of the channel, m
r_t	Radius of the tube (m)
s	Specific entropy, J/(kgK)
T	Temperature, K
t	Time, s
t_{dis}	Time period required for disintegration of a bubble, s
t_{res}	Residence time, s
$U = \sqrt{\frac{p}{\rho}}$	Scale velocity, m/s

u	Velocity component, m/s
V	Volume, m^3
v	Specific volume, m^3/kg
W_*	Work for critical nucleus creation, J
w	Velocity component, m/s
w_∞	Rise velocity of a single bubble, m/s
$w_* = \sqrt{\frac{\tau_w}{\rho}}$	Friction velocity, m/s
w_t	Velocity of turbulent moles, m/s
x	Mass quality
$x_\delta = \frac{h_{tp} - h_{lsat}}{h_{fg}}$	Relative enthalpy of the flow
Y	Boundary position, m
y	Transverse coordinate, m
Z	Channel length, m
z	Longitudinal coordinate, m

Greek Symbols

$\alpha = \frac{k_t}{\rho c_p}$	Thermal diffusivity, m^2/s
$\alpha_{\max}$	Amplitude increment of dangerous oscillations, 1/s
β	Vapour void ratio
Γ	Gamma-function
$\gamma = \frac{c_p}{c_v}$	Adiabatic index
Δp	Pressure difference, Pa
ΔT	Temperature difference, K
$\Delta T_l = T_{sat} - T_l$	Liquid subcooling, K
$\Delta T_w = T_w - T_{sat}$	Wall superheat, K
Δ_F	Vapour generation rate due to surface sites, kg/(m^3s)
Δ_V	Vapour generation rate due to volume sites, kg/(m^3s)
ε_{in}	Jet contraction ratio
$\varepsilon = \frac{\rho_v}{\rho_l}$	Density ratio
ϑ	"Age" of the surface element, s
κ	Politropic exponent
λ	Wavelength, m
μ	Dynamic viscosity, Pas
μ_∞	Flow coefficient
μ_{in}	Flow coefficient for the channel entrance
$v = \mu/\rho$	Kinematic viscosity, m^2/s
$\xi = \frac{r}{R}$	Dimensionless coordinate
ρ	Density, kg/m^3
σ	Surface tension, N/m
τ	Share stress, Pa

Φ	Surface “age” distribution function, 1/s
ϕ	Chemical potential, J/kg
φ	Void fraction
φ_{in}	Velocity coefficient in the contracted section
$\chi = 1 - \xi$	Dimensionless parameter
$\omega = 2\pi f$	Angular frequency, 1/s

Subscripts

0	Bulk of still liquid, channel entrance
∞	Infinity
b	Bubble
cr	Critical
eq	Thermal equilibrium
g	Gas
l	Liquid
max	Maximal value
min	Minimal value
n	Normal component
s	Isentropic
sat	Saturation line
sp	Spinodal
tp	Two-phase
v	Vapour
w	Wall

Diacritic Signs and Superscripts

$\dot{O} = \frac{\mathrm{d}o}{\mathrm{d}t}$	Time derivative
$\ddot{O} = \frac{\mathrm{d}^2 o}{\mathrm{d}t^2}$	Second time derivative
$\vec{O}$	Vector
$\bar{O}$	Mean value
O'	Saturated liquid curve
O''	Saturated vapour curve

Definition of Similarity Numbers and Nondimensional Groups

$\mathrm{A} = \frac{r_0 j c_p \Delta T_l}{k_t \Delta T_w}$	Similarity number characterizing the relative role of convective and conductive mechanisms of heat transfer in flows with “double disequilibrium”
$\mathrm{Bo} = \frac{g(\rho_l - \rho_v) d_V^2}{\sigma}$	Bond (Eötvös) number

$\mathrm{Fo} = \frac{\alpha t}{R^2}$	Fourier number
$\mathrm{Fr} = \frac{\rho_l w_\infty^2}{g(\rho_l - \rho_v) d_V}$	Froude number
$\mathrm{Ja} = \frac{\rho_l c_{pl} \Delta T}{\rho_v h_{fg}}$	Jakob number
$\mathrm{Ka} = \frac{\sigma^3 \rho_l^2}{g(\rho_l - \rho_v) \mu_l^4}$	Kapitsa number
$K_p = \frac{p_0}{p_\infty}$	Pressure ratio
$\mathrm{Mo} = 1/\mathrm{Ka}$	Morton number
$\mathrm{N}_{\mu\sigma} = \frac{\mu_l w_\infty}{\sigma}$	Viscous-capillary criterion
$\mathrm{Nu} = \frac{k_q d}{k_t}$	Nusselt number
$\mathrm{Nu}_D = \frac{k_m d}{k_t}$	Diffusion Nusselt number
$\mathrm{Pe} = \frac{wd}{k_q}$	Peclet number
$\mathrm{Pr} = \frac{\nu}{\alpha}$	Prandtl number
$\mathrm{Pr}_D = \frac{\nu}{D}$	Diffusive Prandtl number (Schmidt number)
$\mathrm{Re} = \frac{wd}{\nu}$	Reynolds number
$\mathrm{St} = \frac{q}{\rho_l w c_{pl} \Delta T}$	Stanton number
$\Re = \frac{UR_0}{\alpha}$	Dimensionless bubble radius
$\bar{\Re} = 1 - R_*/R_0$	Complimentary bubble radius
$\mathrm{S} = \frac{c_{pl} \Delta T}{h_{fg}}$	Stefan number
$\mathrm{We} = \frac{\rho_l w_\infty^2 d_V}{\sigma}$	Weber number
$\Omega = \frac{\omega R_0}{U}$	Dimensionless circular frequency of bubble pulsations
$\tilde{\theta} = T/T_{cr}$	Reduced temperature
$\tilde{\pi} = p/p_{cr}$	Reduced pressure
$\tilde{\omega} = v/v_{cr}$	Reduced specific volume

Chapter 1
Introduction. General Principles of Description of Two-Phase Systems

1.1 Two-Phase Systems

Any substance of uniform chemical composition may be found in various *phases*, which are different states of aggregation (gas, liquid, solid, and plasma). In the solid state, a substance may be also composed of various phases, each of which has the same composition, but different structure (allotropy). The characteristic feature of phases is the presence of boundaries separating a given phase from the different adjacent phases. It is quite natural that the properties of each of the phases are defined by its specific equation of state.

A substance may transform from one phase to another. This transformation is called the *phase transition*. The transition of a substance from the condensed (solid or liquid) phase into the gas phase is called *evaporation* or vapour generation (the evaporation of solids is sometimes called *sublimation*), the inverse transformation is known as *condensation*. A transition from one solid phase into the liquid one is called *melting*, while the inverse transformation from the liquid phase into the solid one is known as *solidification* or crystallization. Phase transitions are accompanied by emission or absorption of heat, which is called the *heat of phase transition* (the heat of phase transition per unit mass is usually denoted as h_{fg}).

Multiphase systems are systems consisting of two or more phases. If all phases forming a system under consideration consist of one chemical substance, then such a system is called a *single-component system* (respectively, two- or multicomponent if it consists of two or more substances).

Multiphase systems are common both in nature and in engineering. Within the frames of engineering applications, we shall consider, as examples, the generation and subsequent condensation of vapour in power generating units; the processes of distillation, rectification, evaporation playing a significant role in chemical engineering, cryogenic and refrigerating machinery, as well as in food production. In practice, various types of multiphase systems (bubble flows, aerosols, liquid emulsions, flows of liquid and gas with solid inclusions and so on) are more common than

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